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CALCULATION OF STRESS AT BENDING OF A BEAM OF POLYMERIC MATERIAL

Purpose. *The purpose of research is to obtain a mathematical model that describes the dependence of stress and strain at bending whole and hollow polymeric beams of round and rectangular cross section.*

Methodology. *Research methodology is based on the analysis of the bending of a beam from polymeric material stress distribution is determined along the height of its cross section.*

Findings. *Expressions for calculation of whole and hollow beams of polymeric material for durability are obtained.*

Originality. *The mathematical model of bend of polymer beams with non-linear relationship between the strain and the stress.*

Practical value. *The results can be used in the design of structural elements of machines and appliances, building constructions, products of light industry.*

Keywords: *strength, deformation, bending, polymeric beam.*

Introduction. Polymeric materials are widely used in various fields of industrial production for manufacturing machine parts, structural elements of buildings, light industrial products [1-4]. Therefore strength and stiffness of such products with the peculiarities of their behavior under operational loads become very important. It is found previously that the physical and mechanical properties of polymer materials do not provide dependency between the strains and stresses according to the Hooke's law. The authors of [5] use the power dependence, which can be put in the basis for calculating the strength and deformability of details from polymeric materials.

Problem definition. In the work [5] the expression is obtained to determine the stress in the beam from a polymeric material during its bending:

$$\sigma = \frac{M z^{\frac{1}{m}}}{I_m}, \tag{1}$$

where σ – stress; M – bending moment; z – vertical coordinate.

$$I_m = \int_A z^{\frac{1}{m}+1} dA. \tag{2}$$

For beams of rectangular cross sections the value of I_m is determined as:

$$I_m = \frac{2b}{\frac{1}{m}+2}\left(\frac{h}{2}\right)^{\frac{1}{m}+2}. \tag{3}$$

It is important to determine stresses and deformations in bending whole and hollow beams of circular and rectangular cross section from polymeric materials.

Results of a research. For the cantilever beam with the applied force to its free end (Fig. 1) the bending moment is defined as:

$$M = P(l - x). \tag{4}$$

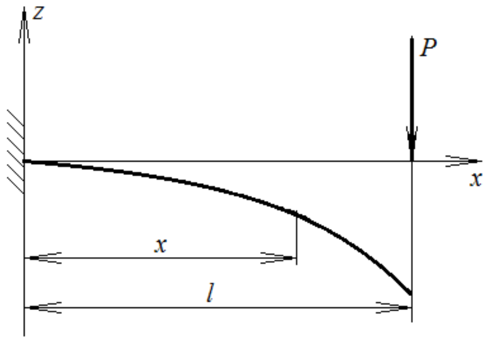

Fig. 1. **Scheme of a loaded cantilevered beam from a polymeric material**

The largest bending moment will be:

$$M = Pl. \tag{5}$$

Substituting (3) and (4) into (1) we get:

$$\sigma = \frac{Pl \cdot \left(\frac{1}{m} + 2\right)}{2 \cdot b \cdot \left(\frac{h}{2}\right)^2}. \tag{6}$$

The estimated the height of beam's cross-section:

$$h = 2\sqrt{\frac{Pl \cdot \left(\frac{1}{m} + 2\right)}{2 \cdot b \cdot [\sigma]}}. \tag{7}$$

Taking into account the allowable value of stresses the dependence of the height of beam's cross-section form its length can be calculated to ensure even strength along longitudinal coordinate:

$$h = 2\sqrt{\frac{P(l - x)\left(\frac{1}{m} + 2\right)}{2 \cdot b \cdot [\sigma]}}. \tag{8}$$

Fig. 2 shows graphic dependencies calculated by the expressions (1) and (3) for beams of square cross-section with a side measured 6 mm.

Fig. 3 shows the dependence of the height of cross-section for a beam with width measured 6 mm from the longitudinal coordinate, calculated using the expression (8).

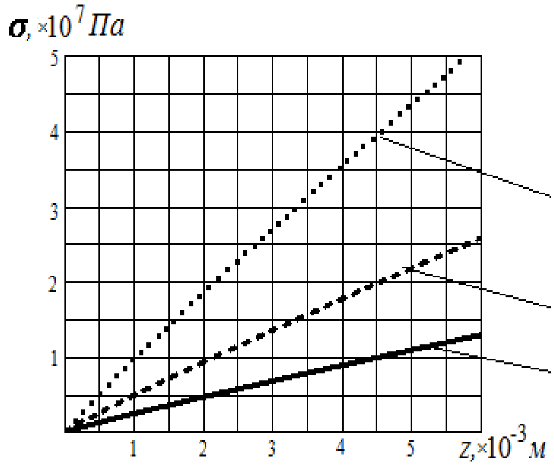

Fig. 2. **Dependence of the stress on the vertical coordinate for square beam cross section**: *1 –* $M = 0.25\, N{\cdot}m$; *2 –* $M = 0.5\, N{\cdot}m$; *3 –* $M = 1 N{\cdot}m$

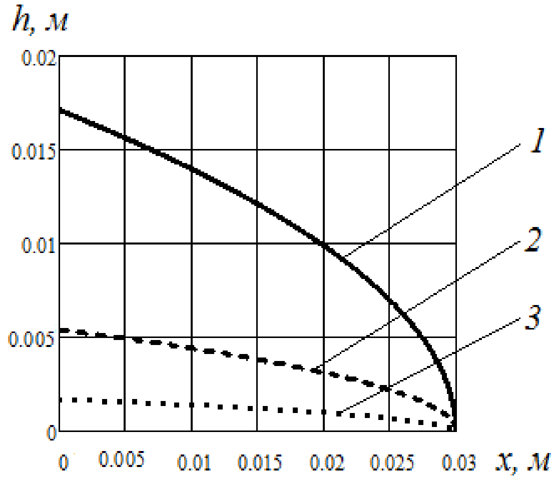

Fig. 3. **Dependence of cross-section height for a cantilevered beam from the longitudinal coordinate**: *1 –* $[\sigma] = 10^5\, Pa$; *2 –* $[\sigma] = 10^6\, Pa$; *3 –* $[\sigma] = 10^7\, Pa$

For hollow beams of rectangular cross-sections it is obtained [5]:

$$I_m = \frac{2m}{1+2m}\left[B\left(\frac{H}{2}\right)^{\frac{1}{m}+2} - b\left(\frac{h}{2}\right)^{\frac{1}{m}+2}\right]. \tag{9}$$

Substituting (9) and (5) into (1) for hollow beams we get:

$$\sigma = \frac{Pl\cdot(1+2m)}{2bm\left(\frac{h}{2}\right)^2\left[\frac{B}{b}\left(\frac{H}{h}\right)^{\frac{1}{m}+2} - 1\right]}. \tag{10}$$

From (10) the estimated height of the beam will be:

$$h = 2\sqrt{\frac{Pl\cdot(1+2m)}{2\cdot b\cdot m\cdot[\sigma]\cdot\left[\frac{B}{b}\left(\frac{H}{h}\right)^{\frac{1}{m}+2} - 1\right]}}. \tag{11}$$

For a beam of circular cross-section Fig. 4 we get:

$$I_m = \int_A z^{\frac{1}{m}+1} dA = 2\int_0^r z^{\frac{1}{m}+1} y dz = 2\int_0^r z^{\frac{1}{m}+1}\sqrt{r^2 - z^2}\, dz. \tag{12}$$

Let us introduce substitution:

$$x = r^2 - z^2; \tag{13}$$

$$dx = -2zdz; \tag{14}$$

$$dz = -\frac{dx}{2z} = -\frac{dx}{2x^{\frac{1}{2}}}. \tag{15}$$

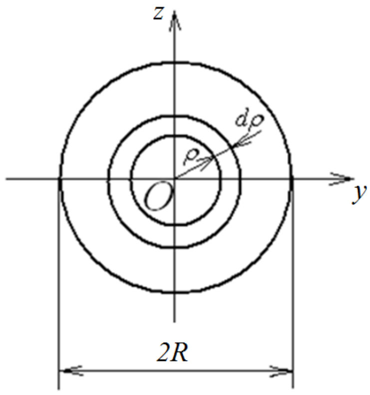

Fig. 4. **Scheme of the beam's circular cross-section**

Then:

$$I_m = -2\int_{r^2}^{0} x^{\frac{\frac{1}{m}+1}{2}} x^{\frac{1}{2}} \frac{dx}{2x^{\frac{1}{2}}} = -\int_{r^2}^{0} x^{\frac{\frac{1}{m}+1}{2}} dx = -\frac{1}{\frac{\frac{1}{m}+1}{2}+1} x^{\frac{\frac{1}{m}+1}{2}+1}\Bigg|_{r^2}^{0} = \frac{2}{\frac{1}{m}+3} r^{\frac{1}{m}+3} \tag{16}$$

For hollow beams we get:

$$I_m = I_m(R) - I_m(r) = \frac{2}{\frac{1}{m}+3}\left(R^{\frac{1}{m}+3} - r^{\frac{1}{m}+3}\right). \tag{17}$$

where $I_m(R)$ and $I_m(r)$ – the values determined for beams with radii R and r respectively. Substituting (16) in (1), for the maximum stress we obtain:

$$\sigma = \frac{Pl\cdot\left(\frac{1}{m}+3\right)}{2\cdot r^3}. \tag{18}$$

From (18) the estimated radius of the beam will be:

$$r = \sqrt[3]{\frac{Pl\cdot\left(\frac{1}{m}+3\right)}{2\cdot[\sigma]}}. \tag{19}$$

Substituting (17) into (1), we get:

$$\sigma = \frac{Pl\cdot\left(\frac{1}{m}+3\right)}{2\cdot R^3\cdot\left(1-\left(\frac{r}{R}\right)^{\frac{1}{m}+3}\right)}. \tag{20}$$

From (20) the estimated radius of the hollow beam will be:

$$R = \sqrt[3]{\frac{Pl \cdot \left(\frac{1}{m}+3\right)}{2 \cdot [\sigma] \cdot \left(1 - f^{\frac{1}{m}+3}\right)}}, \tag{21}$$

where $f = \frac{r}{R}$ – the given ratio.

Conclusions. On the basis of a nonlinear relationship between the stress and the strain derived expressions calculation of durability of beams from polymeric material. Obtained formulae can be used in the process of design of structural elements of machines and appliances, building constructions and products of light industry.

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РОЗРАХУНОК НАПРУЖЕНЬ ПРИ ЗГИНІ БАЛОК ІЗ ПОЛІМЕРНОГО МАТЕРІАЛУ

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Мета. *Отримати математичну модель, що описує залежність напружень та деформацій при згині суцільних та порожнистих балок із полімерних матеріалів круглого та прямокутного поперечного перерізу.*

Методика. *У роботі використано аналітичні методи досліджень. На основі аналізу згину балки із полімерного матеріалу визначено розподіл напружень по висоті її поперечного перерізу.*

Результати. *Отримані вирази для розрахунку суцільних та порожнистих балок із полімерного матеріалу на міцність.*

Наукова новизна. *Розроблено математичну модель згину полімерної балки з нелінійним характером залежності між деформаціями та напруженнями.*

Практична значимість. *Результати можуть бути використані під час проектування конструктивних елементів машин та приладів, будівельних конструкцій, виробів легкої промисловості.*

Ключові слова: *міцність, деформація, згин, полімерна балка.*

РАСЧЕТ НАПРЯЖЕНИЙ ПРИ ИЗГИБЕ БАЛОК С ПОЛИМЕРНЫМИ МАТЕРИАЛАМИ

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Цель. *Получить математическую модель, описывающую зависимость напряжений и деформаций при изгибе сплошных и полых балок из полимерных материалов круглого и прямоугольного поперечного сечения.*

Методика. *В работе использованы аналитические методы исследований. На основе анализа изгиба балки из полимерного материала определено распределение напряжений по высоте ее поперечного сечения.*

Результаты. *Получены выражения для расчета сплошных и полых балок из полимерного материала на прочность.*

Научная новизна. *Разработана математическая модель изгиба полимерной балки с нелинейным характером зависимости между деформациями и напряжениями.*

Практическая значимость. *Результаты могут быть использованы при проектировании конструктивных элементов машин и приборов, строительных конструкций, изделий легкой промышленности.*

Ключевые слова: *прочность, деформация, изгиб, полимерная балка.*