COMPUTER SIMULATION OF MULTIPLE MEASUREMENTS OF LOGARITHMIC TRANSFORMATION FUNCTION BY TWO APPROACHES

The studies of the capabilities of redundant measurement methods revealed the high efficiency of the presented methods in increasing the accuracy of multiple measurements. It was proved that redundant measurement equations ensure the independence of the measurement result from the parameters of the transformation function and their deviations from the nominal values. Experimental studies have confirmed that the accuracy of multiple measurements is increased by processing the results of intermediate measurements using equations of redundant measurements by two approaches. In particular, it was found that processing the results of multiple measurements with the logarithmic transformation function with the first approach provides the value of the relative error of $0.75 \cdot 10^{-3}$ %, and the second – $0.02 \cdot 10^{-3}$ %. This suggests that the increase in accuracy is due to the total effect of the elimination of the systematic error component due to changes in the parameters of the transformation function and reduction of the random error component. The latter, in particular, concerns the algorithms for processing multiple measurements by two approaches. A comparative analysis was made, the advantages and disadvantages of each of the two approaches were determined. It was found that the second approach is less sensitive to an increase in the difference between the values of the controlled and normalized quantities. This allows us to state the possibility of measuring the controlled parameter (F_x) of a large value without imposing high requirements on the power of the calibrated radiation source.

There is reason to assert about the promising development of redundant measurement methods in the processing of the results of multiple measurements in the field of increasing accuracy with the nonlinear transformation function

Keywords: redundant methods, multiple measurements, measurement equation, function parameters, accuracy increase

V. Shcherban'
Doctor of Technical Sciences, Professor, Head of Department*
E-mail: shherban.vy@knutd.com.ua

G. Korogod
PhD, Associate Professor*
E-mail: 5618607@ukr.net

O. Kolysko
PhD, Associate Professor*
E-mail: kolisko.oz@knutd.com.ua

M. Kolysko
PhD, Associate Professor*
E-mail: kolisko.oz@knutd.com.ua

Yu. Shcherban'
Doctor of Technical Sciences, Professor, Head of Department**
E-mail: scherban@i.ua

G. Shchutska
Doctor of Technical Sciences, Associate Professor, Director**
E-mail: polischuka.kklp@gmail.com
*Department of Computer Science and Technologies
Kyiv National University of Technologies and Design
Nemyrovycha-Danchenka str., 2, Kyiv, Ukraine, 01011
**Department of Light Industry Technologies
State Higher Educational Establishment
«Kyiv College of Light Industry»
Dzhona Makkeina str., 29, Kyiv, Ukraine, 01601

1. Introduction

Modern industrial development poses urgent tasks for scientists and engineers to improve measurement accuracy and obtain reliable information (about the controlled parameter or object of study) while reducing the cost of metrological support. This task is especially acute when performing technological processes, such as in the chemical or textile industry. This is because measurement error or inaccuracy of the information obtained can lead to the release of poor-quality product [1–3] or its lack, and sometimes even delay the entire process. The accuracy of the measuring instrument and the improvement of the relevant measurement methods play a special role in resolving this issue. As is known, the main element of the measuring means is a sensor that converts the input controlled physical quantity, characterizing the technological process or property of the object under study, into a proportional electrical output signal. The accuracy of the whole subsequent measuring process depends on the accuracy of this transformation. Because even a small deviation

introduced by the sensor will only be amplified during further processing in the measured channel. Therefore, sensors have the following requirements: high accuracy, high sensitivity, wide operating range, stability, etc. In addition, the type of sensor transformation function also affects the measurement accuracy. Thus, for the nonlinear transformation function, it is necessary to carry out linearization, which causes a nonlinearity error, or to narrow the operating range and work on linear sections of the input characteristic.

Particular attention should be paid to the industries and processes where long-term multiple measurements are needed to control the measuring parameter (quantity) within the tolerance. With such a long-term measurement control under the influence of the environment, the parameters of the sensor transformation function change, which affects the accuracy of the measurement result [4] and, as a consequence, product quality. Thus, in multiple measurements, on the one hand, it is necessary to adhere to high measurement accuracy with the nonlinear transformation function, and on the other – minimize the need for frequent sensor calibration by changing the parameters of the transformation function.

Therefore, studies aimed at improving the accuracy of multiple measurements with the nonlinear transformation function should be considered relevant.

2. Literature review and problem statement

Technical progress encourages in-depth study and analysis of the causes of measurement errors, as well as ways to overcome them. In [5], increased accuracy was achieved using additive and multiplicative tests. It is shown that the additive test is implemented by introducing a reference standard, and multiplicative – by a reference change in the sensitivity of the measuring channel. However, when using the proposed approach, it is necessary to strictly adhere to the ratio of the reference weight and gain values. Also, the issue of processing measurement results under instability and nonlinearity of the transformation characteristics remains unresolved. In [6], the increase in measurement accuracy and sensitivity was achieved by reducing the influence of dark current and compensation of the own noise of the measuring channel of the radiometric instrument. It is shown that due to the switching of the signals of the reference (darkened) and measuring (open) photodiodes and their processing, the photodetector noises are eliminated and the influence of their dark currents is reduced. The issue of increasing the measurement accuracy by increasing the photodetector sensitivity was also considered in [7]. However, it should be noted that these works do not take into account the effect of changes in the parameters of the transformation function on the measurement result, which, in turn, can lead to an increase in measurement error. This is due to the influence of the environment on the measuring channel with the sensor, as well as material aging. To overcome this problem, the works [8, 9] give appropriate calculation algorithms, which resulted in the improvement of the output sensor signal under different conditions. However, these works do not define exactly how to perform calculations for the nonlinear transformation function with unstable parameters, which, in turn, are components of systematic error. Theoretical aspects of the systematic and random components of measurement error [10] and some ways to reduce them were considered in [11]. Studies have shown

that the minimization of the systematic error component was proposed to be achieved by developing and formulating appropriate requirements and measures to reduce systematic error and its components – instrumental, methodological and subjective. Despite the importance of such approaches, ways to reduce the random component of measurement error have not been sufficiently considered. The main uncertainties and their assessment were considered in [12]. It is shown that by estimating these factors, it becomes possible to obtain more accurate measurement results of optical radiation, or, as noted in [13], other controlled parameter. In [14], an increase in accuracy and the reduction of sensor uncertainty were achieved by the aggregation of data in a multidimensional sensor. However, these works did not address the issue of direct work with the nonlinear transformation function. This is due to the fact that linearization of the transformation function causes a nonlinearity error or requires working on linear sections of the input sensor characteristic, which narrows the measurement range. In [15], the expansion of the range was achieved by using programmed gain factors and synchronous detectors. However, it should be noted that these works do not provide ways to improve the accuracy with nonlinear and unstable transformation functions. Accuracy improvement by reducing the effect of nonlinearity was considered in [16]. Here, by calibration at several reference temperatures, a reduction in the correction curvature and, thus, an increase in accuracy were achieved. However, elimination of the influence of the systematic error component, caused by the instability of the parameters of the transformation function under the influence of external factors, on the measurement result is not considered.

Improving the measurement accuracy with nonlinear and unstable transformation functions using redundancy was considered in [17–19]. However, approaches to processing the results of multiple measurements with the nonlinear (logarithmic) transformation function have not been studied. This is due to the fact that multiple measurements are known to reduce the impact of the random component of measurement error and, as a consequence, lead to improved measurement reliability.

Therefore, there is reason to believe that the lack of certainty in studies of multiple measurements with the nonlinear transformation function using redundancy necessitates research in this direction.

3. The aim and objectives of the study

The study aimed to determine the patterns and features of applying multiple measurements with the logarithmic transformation function by two approaches that improve measurement accuracy.

To achieve the aim, the following objectives were set:
– to develop mathematical models of multiple measurements with the logarithmic transformation function by two approaches;
– to carry out a computer simulation of the proposed mathematical model by two approaches with changes in the transformation function parameters from ±1.0 % to ±10.0 %, as well as with the reduced (two and four times) value of the normalized flow;
– to make a comparative analysis of the proposed approaches.

4. Materials and methods of the study of computer simulation of redundant measurement methods

4. 1. Materials and simulation tools

The studies were performed using a PD307 silicon photodiode, which has the logarithmic transformation function (TF) and the following parameters: dark current of the photodiode $Is = 0.003\,\mu A$, current (monochrome) sensitivity $S_{I\lambda} = 0.27$ A/W (at $\lambda = 0.55\,\mu m$).

Mathcad 15 software was chosen for mathematical simulation and analysis of sensor behavior with different input data.

4. 2. Method of the study of redundant measurements

As is known [20], the transformation function of a photodiode in photovoltaic mode (with load) has the form:

$$U_R = \frac{kT}{q}\ln\left(\frac{S_I F_x}{I_S} + 1\right) - U_{RM},\qquad(1)$$

where U_R – the load voltage; k – Boltzmann constant ($k = 1.38 \cdot 10^{-23}$ J/K); T – photodiode temperature (ambient temperature or temperature stabilization unit); q – electron charge ($q = 1.6 \cdot 10^{-19}$ Cl); S_I – current (monochromatic) sensitivity of the photodiode; F_x – flux of optical radiation incident on the photodiode; Is – dark current of the photodiode; U_{RM} – voltage drop across the ohmic elements of the diode.

To simplify the expression (1), the following substitutions are introduced: $kT/q = S'_H$ (where S'_H – steepness of transformation) and $S_I/I_S = 1/F_T$ (where F_T – dark flow). As a result of the proposed substitutions, expression (1) will take the form:

$$U'_R = S'_H \ln\left(\frac{F_x}{F_T} + 1\right) - U'_{RM}.\qquad(2)$$

Since the controlled quantity in equation (1) is the flow F_x, equation (2) was solved with respect to it. As a result, the following expression was obtained:

$$F_x = F_T\left(e^{\frac{U'_R + U'_{RM}}{S'_H}} - 1\right).\qquad(3)$$

In equations of quantities (2) and (3), the parameters U'_{RM} and S'_H are indicated with dashes, meaning their real (not ideal) values, i. e. values with errors.

Therefore, when using non-redundant methods, the result of measuring the controlled quantity of F_x will depend on the exact knowledge of these quantities. Also, it is necessary to linearize the logarithmic transformation function or work on its linear section. The tasks of reducing the nonlinearity error and instability of the transformation function are successfully solved by redundant measurement methods (RMM).

To implement redundant measurements, it is necessary to compile a mathematical model as a system of equations describing measurement cycles. In this case, the number of cycles is determined by the number of variables in the presented TF. Since the TF described by equation (2) has 4 variables, the system of equations will consist of 4 equations of quantities. For this purpose, in addition to the measurement cycle of the controlled quantity F_x, measurement cycles of the normalized radiation fluxes F_0 and ΔF_0 are performed. These flows are formed using standard sources with normalized characteristics. Thus, the photodiode sequentially (in different measurement cycles) receives 4 streams of radiation. In the first cycle, the photodiode receives a normalized radiation flux F_0, in the second – two normalized fluxes F_0 and ΔF_0 simultaneously. In the third cycle, only the controlled quantity F_x is measured, and in the fourth – the controlled quantity F_x and normalized quantity of the radiation flux ΔF_0 simultaneously. As a result, we obtain the following system of equations of quantities, consisting of four measurement cycles:

$$\begin{cases} U'_{R1} = S'_H \ln\left(F_0/F_T + 1\right) - U'_{RM}, \\ U'_{R2} = S'_H \ln\left((F_0 + \Delta F_0)/F_T + 1\right) - U'_{RM}, \\ U'_{R3} = S'_H \ln\left((F_x)/F_T + 1\right) - U'_{RM}, \\ U'_{R4} = S'_H \ln\left((F_x + \Delta F_0)/F_T + 1\right) - U'_{RM}, \end{cases}\qquad(4)$$

where U'_{Ri} – voltage in each i-th ($i = (1 \div 4)$) measurement cycle.

It should be noted that all i measurement cycles (for the logarithmic transformation function $i = (1 \div 4)$) make up one measurement cycle.

As a result of solving this system with respect to the controlled quantity F_x, we obtain the expression:

$$F_x = \frac{\Delta F_0}{\left[\left(\Delta F_0 \big/ \left((F_0 + F_T) + 1\right)\right)^{(U'_{R4} - U'_{R3})/(U'_{R2} - U'_{R1})}\right] - 1} - F_T.\qquad(5)$$

The resulting equation (5), in contrast to equation (3), does not include the parameters U'_{RM} and S'_H, that is, the result of measuring the controlled quantity F_x does not depend on the TF parameters and their changes. Thus, RMM help to eliminate the systematic error component caused by the instability of the transformation function parameters. It should be noted that this result is achieved provided that changes in the parameters remain constant during the measurement cycle.

To check the correctness of the obtained equation (5), it is enough to substitute expressions from the system (4) instead of current voltages U'_{Ri}. As a result, we obtain the redundant measurement equation:

$$F_x = \frac{\Delta F_0}{\left[\left(\Delta F_0 \big/ \left((F_0 + F_T) + 1\right)\right)^{\frac{(S'_H \ln((\Phi_x + \Delta\Phi_T)/\Phi_T + 1) - U'_{RM} - S'_H \ln((\Phi_0 + \Delta\Phi_T)/\Phi_T + 1) - U'_{RM})}{(S'_H \ln((\Phi_0 + \Delta\Phi_T)/\Phi_T + 1) - U'_{RM} - S'_H \ln((\Phi_0/\Phi_T) + 1) - U'_{RM})}}\right] - 1} - F_T = F_x.\qquad(6)$$

As can be seen from the redundant measurement equation (6), the value of the controlled parameter F_x is brought to the input. This means that when using RMM, no additional linearization of the logarithmic transformation function is required.

4. 3. Method of the study of RMM with multiple measurements

As is known, in the case of multiple measurements, when random error components should be taken into account, statistical data processing methods are commonly used. In the theory of redundant measurements, this procedure is performed by two approaches differing in the algorithm for processing the results of multiple measurements [21, 22]. With the first approach, the voltages for each cycle are measured. After the m cycle, the obtained voltage results are averaged over each

of the cycles, and the obtained average values are substituted into the corresponding equation of redundant measurements of the controlled physical quantity. With the second approach, the values of the controlled physical quantity in each cycle are first found by the redundant measurement equation, and then the obtained values are averaged over m measurement cycles.

The application of these two approaches with the logarithmic transformation function based on the system of equations (4) and redundant measurement equation (5) was considered.

With the first approach, the current voltage values U'_{R1}, U'_{R2}, U'_{R3}, U'_{R4} are measured in each cycle according to the system of equations (4). Upon completion of m measurement cycles, the voltage values are averaged over each cycle, and then the obtained average voltage values $\bar{U}'_{R1}$, $\bar{U}'_{R2}$, $\bar{U}'_{R3}$, $\bar{U}'_{R4}$ are substituted in the redundant measurement equation (5). As a result, the following expression is obtained:

$$F_x = \frac{\Delta F_0}{\left[\left(\Delta F_0 / \left((F_0 + F_T)+1\right)\right)^{\left(\frac{\sum_{i=1}^{m} U'_{R4i}}{m} - \frac{\sum_{i=1}^{m} U'_{R3i}}{m}\right) / \left(\frac{\sum_{i=1}^{m} U'_{R2i}}{m} - \frac{\sum_{i=1}^{m} U'_{R1i}}{m}\right)} - 1\right]} - F_T. \quad (7)$$

With the second approach, the controlled quantity F_x is measured in each cycle by the redundant measurement equation (5). Upon completion of m measurement cycles, the current values of the controlled quantity are averaged, which will be considered as the measurement result. Thus, the redundant measurement equation (5) by the second approach will take the form:

$$F_x = \frac{\sum_{i=1}^{m} F_{xi}}{m} = \\ = \frac{\sum_{i=1}^{m} \frac{\Delta F_0}{\left[\left(\Delta F_0 / \left((F_0 + F_T)+1\right)\right)^{(U'_{R4i} - U'_{R3i})/(U'_{R2i} - U'_{R1i})} - 1\right]} - F_T}{m}. \quad (8)$$

Based on the presented mathematical models, the computer simulation of these approaches for their further analysis and research was conducted.

5. Results of computer simulation of two approaches

5. 1. Computer simulation of two approaches and their results with a given value of the normalized flow

The computer simulation was performed in Mathcad 15, with the following parameters of the FD307 photodiode: voltage drop across the ohmic elements of the diode $U_{RM}=0.01$ V, dark current of the photodiode $I_s=0.003$ µA, current (monochromatic) sensitivity $S_{I\lambda}=0.27$ A/W (at $\lambda=0.55$ µm). Based on the proposed replacement $\left(S_I/I_s = 1/F_T\right)$, the value of the dark flux $F_T=1.111 \cdot 10^{-8}$ W was obtained. The operating point is set to the flux value $F_x=0.001$ W. Because the values of normalized physical quantities in the sum must be of the same order as the controlled one, $F_0=0.8$ mW, $\Delta F_0=0.1$ mW were chosen.

The values of the reproduction error of the normalized radiation fluxes F_0 and ΔF_0 were also proposed, which will be ±0.1 %. This low error is due to the fact that the normalized quantities F_0 and ΔF_0 are formed using a high-precision calibrated source. Limits for the parameters $\bar{U}'_{RM}$ and S'_H are set in the range from ±1.0 % to ±10.0 %.

As a result of computer simulation of multiple measurements by two approaches, the values presented in Table 1 were obtained. In this case, the values of the TF parameters U'_{RM} and S'_H and their deviations should remain unchanged during one measurement cycle.

It was determined that the relative measurement error of the controlled flow F_x in each measurement cycle will be 0.013 %, which confirms the high accuracy of RMM.

Following the proposed algorithm for processing the results of the first approach, average voltage values for each measurement cycle are first found: $\bar{U}'_1 = 0.2724$ V, $\bar{U}'_2 = 0.2754$ V, $\bar{U}'_3 = 0.2780$ V, $\bar{U}'_4 = 0.2804$ V.

Table 1

Measurement results under changes in parameters U'_{RM} and S'_H

No.	Changes in parameters U'_{RM} and S'_H	U'_{R1}	U'_{R2}	U'_{R3}	U'_{R4}	F_x
1	$U'_{RM} = (0.01 + 0.0001)$ V and $S'_H = S_H \cdot (1+0.01)$	0.2753	0.2783	0.2810	0.2834	$9.998654 \cdot 10^{-4}$
2	$U'_{RM} = (0.01 - 0.0001)$ V and $S'_H = S_H \cdot (1-0.01)$	0.2695	0.2724	0.2750	0.2774	$1.000135 \cdot 10^{-3}$
3	$U'_{RM} = (0.01 + 0.001)$ V and $S'_H = S_H \cdot (1+0.1)$	0.3016	0.3049	0.3078	0.3105	$9.998654 \cdot 10^{-4}$
4	$U'_{RM} = (0.01 - 0.0001)$ V and $S'_H = S_H \cdot (1-0.1)$	0.2432	0.2458	0.2482	0.2504	$1.000135 \cdot 10^{-3}$
6	$U'_{RM} = (0.01 + 0.001)$ V and $S'_H = S_H \cdot (1+0.01)$	0.2762	0.2792	0.2819	0.2843	$9.998654 \cdot 10^{-4}$
7	$U'_{RM} = (0.01 - 0.001)$ V and $S'_H = S_H \cdot (1+0.01)$	0.2742	0.2772	0.2799	0.2823	$9.998654 \cdot 10^{-4}$
8	$U'_{RM} = (0.01 + 0.001)$ V and $S'_H = S_H \cdot (1-0.01)$	0.2706	0.2735	0.2761	0.2785	$1.000135 \cdot 10^{-3}$
9	$U'_{RM} = (0.01 - 0.001)$ V and $S'_H = S_H \cdot (1-0.01)$	0.2686	0.2715	0.2741	0.2765	$1.000135 \cdot 10^{-3}$
10	$U'_{RM} = (0.01 + 0.0001)$ V and $S'_H = S_H \cdot (1+0.1)$	0.3007	0.3040	0.3069	0.3096	$9.998654 \cdot 10^{-4}$
11	$U'_{RM} = (0.01 - 0.0001)$ V and $S'_H = S_H \cdot (1+0.1)$	0.3005	0.3038	0.3067	0.3094	$9.998654 \cdot 10^{-4}$
12	$U'_{RM} = (0.01 + 0.0001)$ V and $S'_H = S_H \cdot (1-0.1)$	0.2443	0.2469	0.2493	0.2515	$1.000135 \cdot 10^{-3}$
13	$U'_{RM} = (0.01 - 0.0001)$ V and $S'_H = S_H \cdot (1-0.1)$	0.2441	0.2467	0.2491	0.2513	$1.000135 \cdot 10^{-3}$
	Average values	0.2724	0.2754	0.2780	0.2804	$10.000002 \cdot 10^{-4}$

The obtained values are substituted in equation (7), resulting in the value of the controlled radiation flux $F_x =$ $= 9.999925 \cdot 10^{-4}$ W and the corresponding value of the relative error δ will be $0.75 \cdot 10^{-3}$ %.

When analyzing the impact of changes in the parameters U'_{RM} and S'_H on the current voltage values, it should be noted that a greater influence is shown by the parameter S'_H. Thus, with an asymmetric change in the parameters U'_{RM} and S'_H during the measurement cycles, it was found that the greater the deviation of the parameter S'_H, the greater the scatter of voltage values relative to their average values.

With the second approach, the current value of the radiation flux F_x was determined for each measurement cycle in accordance with the redundant measurement equation (5). Then, according to equation (8), the obtained values were averaged, which allowed determining the value of the controlled radiation flux $F_x = 1.0000002 \cdot 10^{-3}$ W. It was found that the value of the relative error δ with the second approach will be $0.02 \cdot 10^{-3}$ %.

When considering the impact of changes in the parameters U'_{RM} and S'_H on the current voltage values, it should be noted that the second approach is less sensitive to their changes than the first one.

5. 2. Computer simulation of two approaches and their results when reducing the value of the normalized flow

In this section, the influence of the value of the normalized physical quantity F_0 on the measurement result was considered. To do this, we investigate the case when the difference between the values of the controlled and normalized quantities was increased. The value of the normalized flux F_0 was reduced twice – from 0.8 mW to 0.4 mW. In this case, the previously selected values of other quantities were left unchanged. The results of the computer simulation are presented in Table 2. Table 2 shows the results obtained with the maximum deviations of the parameters U'_{RM} and S'_H.

Thus, when processing the results of the first approach, the value of the controlled quantity of the radiation flux F_x was obtained, which will be $9.999836 \cdot 10^{-4}$ W and, accord-

ingly, the value of the relative error δ will be $1.64 \cdot 10^{-3}$ %. Therefore, with the first approach, an increase in the difference between the values of the controlled and normalized quantities increases the relative error almost 2 times (from $0.75 \cdot 10^{-3}$ % to $1.64 \cdot 10^{-3}$ %).

With the second approach, computer simulation showed that the value of the controlled quantity of the radiation flux will be $F_x = 1.0000001 \cdot 10^{-3}$ W and, accordingly, the value of the relative error δ will be $0.01 \cdot 10^{-3}$ %. Therefore, with the second approach, an increase in the difference between the values of the controlled and normalized quantities does not significantly increase the relative error.

It should be noted that the relative measurement error of the controlled flow F_x in each measurement cycle will be 0.03 %.

We also consider the case when the value of the normalized flux F_0 was reduced four times – from 0.8 mW to 0.2 mW. In this case, the previously selected values of other quantities were left unchanged. The results of computer simulation are presented in Table 3.

As a result, it was obtained that with the first approach, the value of the relative error δ increases from $0.75 \cdot 10^{-3}$ % (at $F_0 = 0.8$ mW) to 0.003 %, and the value of the relative error δ with the second approach will be $0.03 \cdot 10^{-3}$ %.

It should be noted that the relative measurement error of the controlled flow F_x in each measurement cycle will be 0.057 %.

5. 3. Comparative analysis of the proposed approaches

When considering the effect of changes in the parameters on the measurement result, the second approach was less sensitive to their changes than the first one. Thus, the relative error δ with the first approach is $0.75 \cdot 10^{-3}$ %, and with the second – $0.02 \cdot 10^{-3}$ %. The peculiarity of the second approach was that the current value of the radiation flux F_x depends only on the sign of the parameter S'_H and does not depend on the magnitude of its change. It should also be noted that the relative measurement error of the controlled flow F_x in each measurement cycle will be 0.013 %. This suggests that the processing of multiple measurements by two approaches helps to increase accuracy.

Table 2

Measurement results under changes in parameters U'_{RM} and S'_H

No.	Changes in parameters U'_{RM} and S'_H	U'_1	U'_2	U'_3	U'_4	F_x
1	$U'_{RM} = (0.01 + 0.0001)$ V and $S'_H = S_H \cdot (1 + 0.01)$	0.2576	0.2633	0.2810	0.2834	$9.997049 \cdot 10^{-4}$
2	$U'_{RM} = (0.01 - 0.0001)$ V and $S'_H = S_H \cdot (1 - 0.01)$	0.2521	0.2577	0.2750	0.2774	$1.000295 \cdot 10^{-3}$
3	$U'_{RM} = (0.01 + 0.001)$ V and $S'_H = S_H \cdot (1 + 0.1)$	0.2824	0.2886	0.3078	0.3105	$9.997049 \cdot 10^{-4}$
4	$U'_{RM} = (0.01 - 0.001)$ V and $S'_H = S_H \cdot (1 - 0.1)$	0.2274	0.2325	0.2482	0.2504	$1.000295 \cdot 10^{-3}$
Average values		0.2605	0.2652	0.2780	0.2804	$1.0000001 \cdot 10^{-3}$

Table 3

Measurement results under changes in parameters U'_{RM} and S'_H

No.	Changes in parameters U'_{RM} and S'_H	U'_1	U'_2	U'_3	U'_4	F_x
1	$U'_{RM} = (0.01 + 0.0001)$ V and $S'_H = S_H \cdot (1 + 0.01)$	0.2400	0.2503	0.2810	0.2834	$9.99413 \cdot 10^{-4}$
2	$U'_{RM} = (0.01 - 0.0001)$ V and $S'_H = S_H \cdot (1 - 0.01)$	0.2348	0.2449	0.2750	0.2774	$1.000569 \cdot 10^{-3}$
3	$U'_{RM} = (0.01 + 0.001)$ V and $S'_H = S_H \cdot (1 + 0.1)$	0.2631	0.2744	0.3078	0.3105	$9.99413 \cdot 10^{-4}$
4	$U'_{RM} = (0.01 - 0.001)$ V and $S'_H = S_H \cdot (1 - 0.1)$	0.2116	0.2209	0.2482	0.2504	$1.000569 \cdot 10^{-3}$
Average values		0.2605	0.2652	0.2780	0.2804	$1.0000003 \cdot 10^{-3}$

When considering the effect of the value of the normalized flow F_0 on the measurement result (relative measurement error δ), the data presented in Table 4 were obtained.

Table 4

Measurements results under changes in the value of the normalized flow F_0

F_0, mW	δ with the first approach, %	δ with the second approach, %	δ in each cycle, %
0.8	$0.75 \cdot 10^{-3}$	$0.02 \cdot 10^{-3}$	0.013
0.4	$1.64 \cdot 10^{-3}$	$0.01 \cdot 10^{-3}$	0.030
0.2	0.003	$0.03 \cdot 10^{-3}$	0.057

Therefore, the second approach is less sensitive to an increase in the difference between the values of the controlled and normalized quantities and does not significantly increase the relative error (from $0.02 \cdot 10^{-3}$ % to $0.03 \cdot 10^{-3}$ %). It should also be noted that an increase in the difference between the values of the controlled and normalized quantities increases the error of single measurements from 0.013 % (for $F_0 = 0.8$ mW) to 0.057 % (for $F_0 = 0.2$ mW).

6. Discussion of the results of computer simulation of multiple measurements with two approaches

As a result of the computer simulation, the proposed approaches were found to be highly efficient in processing the results of multiple measurements to eliminate the influence of the parameters of the nonlinear TF on the measurement result. This is possible due to the presented equations of redundant measurements, in which due to the subtraction and division of output voltages, the influence of the additive and multiplicative components of the measurement error, respectively, is eliminated. This does not differ from the practical data presented in [5], the authors of which also achieve an increase in accuracy by reducing the additive and multiplicative components of the error.

In addition, statistical processing of the results of multiple measurements according to the proposed algorithms can reduce the random error component, as noted in [21]. Processing results with multiple measurements allows reducing the relative measurement error in comparison with processing results in one cycle. For example, with the second approach, the relative error is $0.02 \cdot 10^{-3}$ %, and the measurement error of the controlled flow F_x in each measurement cycle will be 0.013 %.

In the comparative analysis of the results of computer simulation of multiple measurements, it was found that the second approach was more accurate. Thus, with the first approach, the value of the relative error δ will be $0.75 \cdot 10^{-3}$ %, and with the second – $0.02 \cdot 10^{-3}$ %. Thus, processing the results of intermediate measurements according to equation (8) shows an order of magnitude lower relative error of multiple measurements than processing the results obtained according to equation (7).

It is known that the increase in accuracy is also affected by sensitivity, as evidenced in [7]. When studying the influence of the value of the parameter S'_H, it was found that the second approach is less sensitive to its changes than the first one. Another feature of the second approach is that the current value of the radiation flux F_x depends only on the sign of the parameter S'_H and does not depend on the magnitude of its change. So, when increasing the parameter from +1 % to +10 %, the current value of the radiation flux remains constant $F_x = 9.998654 \cdot 10^{-4}$ W, and when changing the parameter in the range from −1 % to −10 %, the value of $F_x = 1.000135 \cdot 10^{-3}$ W. The advantage of this is that fewer measurement cycles can be performed, and the disadvantage is that the second approach requires more frequent metrological control of TF parameters than the first one.

In the analysis of the proposed approaches, of particular interest is the study of the influence of the normalized flow quantity on the final result. It was found that the second approach is less sensitive to a decrease in the normalized quantity of the radiation flux. Thus, an increase in the difference between the values of the controlled and normalized quantities does not significantly increase the relative error as with the first approach. Thus, with the first approach, the reduction of the normalized quantity of the radiation flux F_0 from 0.8 mW to 0.2 mW increases the relative error from $0.75 \cdot 10^{-3}$ % to 0.003 %. And with the second approach, such a decrease in the value of the normalized flow quantity will almost not affect the result and the value of the relative error δ will increase slightly from $0.02 \cdot 10^{-3}$ % to $0.03 \cdot 10^{-3}$ %.

So, the obtained results of processing multiple measurements by two approaches allow us to assert the prospects of the second approach as it provides:

a) an acceptable relative error ranging from 0.001 % to 0.003 %;

b) the use of low power of the calibrated radiation source to form a calibrated radiation flux, i.e. the quantity of the normalized radiation flux is not tied to the quantity of the controlled flux;

c) independence from deviations in the parameter S'_H, however, this requires more frequent metrological monitoring.

Such conclusions can be considered expedient from a practical point of view, as they allow justifying the use of RMM in multiple measurements by two approaches in different industries, where high-accuracy measurement control is needed. In addition, by eliminating the influence of changes in the parameters of the transformation function of the photodetector on the measurement result, it becomes possible, if necessary, to replace it with the same type and that on a cheap element base (with low requirements for element stability). In addition, when using the second approach, it becomes possible to measure the controlled flux F_x of large magnitude without imposing requirements for high power of the calibrated radiation source.

From a theoretical point of view, it can be stated that RMM in multiple measurements by two approaches can increase measurement accuracy with the nonlinear and unstable TF, which is one of their advantages. However, it is worth noting that such high accuracy results are obtained when changes in parameters remain constant during each measurement cycle. RMM are also characterized by a methodical measurement error, which is due to the reproduction error of the normalized radiation fluxes. The inability to remove these limitations in this study gives rise to a potentially interesting direction for further research.

7. Conclusions

1. Mathematical models of multiple measurements with the logarithmic transformation function by two approaches,

which describe the state of the measuring channel in time, were developed. These features in data processing will be manifested in a high-accuracy result by reducing the random error component and eliminating the influence of the systematic error component caused by the instability of the TF parameters (under the influence of external destabilizing factors). In addition, when using redundant measurement methods, no linearization of the nonlinear TF is needed, which allows working in the entire operating range without breaking down into linear sections. This allows you to directly process large amounts of data with the logarithmic transformation function.

2. Computer simulation of the redundant measurement method by two approaches, which differ in the order of processing intermediate measurement results, was performed. The studies were conducted at different values of deviations (from ±1.0 % to ±10.0 %) of the parameters of the nonlinear transformation function from their nominal values. It was found that the current value of the radiation flux F_x depends only on the sign of the parameter S'_H and does not depend on the magnitude of its change. Thus, when increasing the parameter from +1 % to +10 %, the current value of the radiation flux remains constant $F_x=9.998654 \cdot 10^{-4}$ W, and when changing the parameter in the range from −1 % to −1 0%, the value of the controlled flow F_x will be $F_x=1.000135 \cdot 10^{-3}$ W.

In the simulation, studies were also conducted on the effect of the value of the normalized radiation flux on the result. It was found that the second approach is less sensitive to a decrease in the value of the normalized radiation flux. Thus, when the flux value F_0 decreases from 0.8 mW to 0.2 mW, the value of the relative error δ will increase slightly from $0.02 \cdot 10^{-3}$ % to $0.03 \cdot 10^{-3}$ %, and with the first approach it will increase from $0.75 \cdot 10^{-3}$ % to 0.003 %.

3. A comparative analysis of measurement errors of both approaches was made. It was found that the second approach was more accurate, which provides a relative measurement error in the range from 0.001 % to 0.003 % (depending on the value of the normalized flow).

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